Comparison of three textbooks published for 8th grade in Iceland

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This study compares characteristics of three Icelandic mathematics textbooks for grade 8, used over a period of 25 years. The textbooks’ structure, content, attitudes to mathematics and mathematical competences were compared and portrayed in the light of official curricula. The purpose of the study is to investigate through the focal point of textbooks the implementation of new curricular policy including new topics as well as mathematical competences and attitudes to mathematics. Our findings show that the textbooks differ considerably with respect to the characteristics studied. The objectives of mathematics formulated in the national curriculum are reflected to a great extent in the most recent textbook Átta – tíu, but the widely used translated textbook, Almenn stærðfræði I, deviates in important ways from the curriculum and much more than the Talnaspegill/Hornalína books that only stayed in use for a short while.

Research has shown that teachers of mathematics in many countries rely heavily on textbooks in their day-to-day teaching for decisions such as what to teach, how to teach it and what exercises to use in the classroom and to assign as homework (Robitaille & Travers, 1992; Pepin & Haggarty, 2001; Remillard, 2000).

The Third international mathematics and science study (TIMSS) differentiates between the intended curriculum, the implemented curriculum and the attained curriculum (Robitaille et al., 1993). Later, the potentially implemented curriculum was added to the model (Schmidt et al., 2001). The intended curriculum is reflected in official intentions, aims and goals, for example the national curriculum. Such documents act as policy directives that schools, principals and teachers are expected to put into practice. Textbooks and other organized resource materials reflect

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the potentially implemented curriculum. It can be said that textbooks translate official curricular policies into concrete pedagogical activities that teachers and students put into practice in the classroom and are thus mediators between policy intention and policy implementation. Strategies, practices and activities in the classroom are the implemented curriculum and student knowledge: ideas, constructs, and schemes are the attained curriculum (Schmidt et al., 1997; Schmidt et al., 2001; Valverde et al., 2002).

TIMSS included a large-scale cross-national analysis of mathematics curricula and textbooks as part of its examination of mathematics education in almost 50 countries. The analysis showed a strong correlation between textbooks and classroom work. Textbooks were used as a source of problems and exercises, as reference books, and as a means of instruction (Howson, 1995). Teachers focused on the content of the book and allocated time accordingly. They find that textbooks serve at times as de facto or surrogate mathematics curricula (Schmidt et al., 1997; Schmidt et al., 2001; Valverde et al., 2002).

In Iceland, Sigurgeirsson’s (1992) study on the role and use of textbooks in primary schools showed that mathematical textbooks strongly influence mathematical teaching: 86% of classroom time in mathematics classes was spent on working with the textbook.

Teachers, parents, pupils and others interested in education often express strong and divergent opinions about the books and their merits. At the same time there has been increased interest in the intended or official curriculum as one of the most critical component of educational policy in Iceland.

This study focuses on the relation between the intended curriculum in Iceland, published in the national curriculum, and the potentially implemented curriculum in mathematics textbooks, all published by an official institution, the National centre for educational materials. During the period from 1987 until 2013, only three mathematics textbooks series were published in Iceland for grades 8 to 10. Two of them were still in wide use in 2013. The study maps the terrain by comparing the three textbook series, intended for grade 8, and portrays them in the light of the national curriculum development. The purpose of the analysis is not to rank the textbooks, but to analyse them objectively and form a basis for an informed comparison and discussion.

Historical background

The first national curriculum for primary schools in Iceland was published in 1929. Goals in mathematical education were given as a short list
of facts that children should learn. For example, children in first grade should know the numbers 1–20 and practise addition and subtraction (Námsskrá fyrir barnaskóla, 1929).

In a draft of a national curriculum of 1948 and a national curriculum of 1960, the mathematical content is very similar to the 1929 curriculum: the four arithmetic operations in whole numbers, fractions and decimals, in addition to proportions in the form of the Rule of Three for age 13 to 15. Teaching and evaluation methods were suggested, emphasizing that teachers must look at the ability and maturity of each student (Drög að námsskrám fyrir barnaskóla og gagnfræðaskóla, 1948; Námsskrá fyrir nemendur á fræðosluskyldualdri, 1960). The next draft to a national curriculum, published in 1970, reflected the influence of the New Math movement, although the content had still hardly changed from 1929 (Námsskrá fyrir nemendur á fræðosluskyldualdri, 1970).

The national curriculum, published in 1989, set a new tone in mathematical education in Iceland. Mathematical education was justified, problem solving and clear reasoning were emphasized, and the content increased in diversity, e.g. by including numbers as a separate topic, logic, probability and statistics. Suggestions on teaching methods and methods of evaluation were more detailed (Aðalnámskrá grunnskóla, 1989).

The national curriculum published in 1999, adhered to the one from 1989 but was still more detailed. It was prescriptive in terms of learning aims and objectives but it gave schools a high degree of autonomy over the implementation. Inclusion of mathematics in compulsory school was justified and its place and purpose described. Pupils were to become acquainted with mathematics as part of the cultural heritage, for use in daily life and work, and as a preparation for further education. The objectives set were pupil-oriented and described what pupils were expected to be capable of and know at specific points in their compulsory school study. Performance expectations and attitudes to mathematics, which partly had also appeared in the 1989 curriculum, were further emphasized in the 1999 national curriculum. Pupils were to be able to

- communicate about mathematics,
- solve various exercises and problems,
- reflect, explain and use mathematical reasoning, and
- link mathematics to daily life and other activities, e.g. work, history, art and entertainment.

The content presented included
- numbers,
- arithmetic, including estimation and approximation,
– ratio, proportions and percentages,
– patterns and algebra,
– geometry, and
– probability and statistics.

Teachers were expected to use diverse methods of teaching and certain
defined core mathematical knowledge should be attained by all pupils,
although each individual should get a chance to learn according to his or
her capability (Aðalnámskrá grunnskóla – stærðfræði, 1999).

The national curricula of 1989 and 1999 were influenced by British,
Scandinavian and American writings, such as the NCTM Standards
(Bjarnadóttir, 2006). A new national curriculum, published in 2007, dif-
fered insignificantly from the one from 1999 and is irrelevant in this
context (Aðalnámskrá grunnskóla – stærðfræði, 2007).

In Iceland, the Ministry of education publishes the National curricu-
lum guide for mathematical education. A public institution, the National
centre for educational materials, is responsible for providing all children
in compulsory schools with teaching materials and it receives budget
appropriation for this purpose. It is relatively expensive for a small nation
to publish satisfactory teaching materials in its own language. Usually,
there can only be a choice between two textbook series for each school
subject. The selection of textbooks for publication is therefore of great
importance. No official body has the task to approve textbooks for use in
schools. Teachers have the responsibility to choose the learning material
that will help them to fulfil the requirements of the national curriculum.
A public institute, Námsmatsstofnun [Educational evaluation institute],
runs national tests in Icelandic and mathematics for grades 4, 7 and 10
every autumn, guiding teachers and school authorities, and informing
them about strengths and deficits (Lög um grunnskóla nr. 91/2008).

In 1988, the publication of new Icelandic mathematics textbooks for
grades 8 to 10 began. This textbook series had partly existed for some
time in experimental form, and was now planned as a full three-year
series. However, this series was never fully completed. The two books
for grade 8 were Talnaspegill (Kristjánsdóttir, Magnúsdóttir, Birgisdót-
tir, & Bjarnadóttir, 1988) and Hornalína (Kristjánsdóttir, Magnúsdóttir,
Birgisdóttir, & Bjarnadóttir, 1989). Teachers complained of an unfamiliar
approach to mathematics, too few exercises and too frequent shifts of
topics. This domestic series was reprinted until 1993 but its use dwindled
gradually after it became clear that volume 6 for grade 10 would not be
published.
During the same period, but before the publication of the 1989 national curriculum, a Swedish series for grades 8 to 10 was translated and published. The book intended for grade 8 was *Almenn stærðfræði I* (Björk et al., 1987).

In the early 2000s, a new Icelandic mathematics textbook series for grades 1 to 10 of compulsory school was prepared and published. The books for grade 8 were *Átta – tíu stærðfræði 1* (Pálsdóttir, & Gunnarsdóttir, 2005) and *Átta – tíu stærðfræði 2* (Pálsdóttir & Gunnarsdóttir, 2006). Teachers’ reactions to that series were mixed. Many welcomed what they considered a new and interesting approach to mathematics. Critics typically faulted the same issues as with *Talnaspegill/Hornalina*, the older domestic series: unfamiliar approach to mathematics, too few exercises and too frequent shifts of topics.

Thus for nearly two decades, the translated *Almenn stærðfræði* (Björk et al., 1987) was the most frequently used textbook series for grades 8 to 10. After the publication of the new Icelandic textbook series in 2005–2007 many schools continued to use the older translated series. A recent study (Bjarnadóttir & Hreinsdóttir, in press) reveals that in the majority of schools, the two series, *Almenn stærðfræði* and *Átta – tíu*, were used together in grade 10 in spring 2014.

Aims and research questions

Educators place a great deal of trust in textbooks, so it is important to assess if their content and instructional focus is consistent with official goals for students learning. The purpose of this study is to investigate

i. The differences and similarities of the three textbooks series published in Iceland and used for grade 8 for about 25 years. Textbooks’ structure (representation of material), content, attitudes to mathematics and expected mathematical competences will be compared.

ii. The implementation of policy through textbooks. To what extent does the content of official curriculum statements in mathematics align with those found in textbooks?

Method

Books from the three series of mathematical textbooks were quantitatively analysed by a method based on measuring area of texts. The TIMSS study framework categories (Valverde et al., 2002) influenced the design to a great extent but a simpler framework was developed to fit the scope.
of the study and to better reflect the Icelandic national curriculum. As in the TIMSS study, the content was broken into units called blocks, based on representation format: instructions, examples, exercises, word problems and suggestions for problem solving. Those blocks were further analysed with respect to content, use of diagrams, the promotion of mathematical competencies and attitudes to mathematics. The TIMSS study analysed blocks by looking at their frequency and sequence (Valverde et al., 2002). In this study, the focus is on the measured area of blocks. The area allocated is a natural indicator of the importance given to different presentations of material.

The books are:

Text I. Almenn stærðfræði I (Björk et al., 1986).

Text II. Talnaspegill and Hornalína (Kristjánsdóttir et al., 1988; 1989).

Text III. Átta – tíu stærðfræði 1 and 2 (Pálsdóttir and Gunnarsdóttir, 2005; 2006).

Procedure

The blocks were first analyzed according to presentation formats and the area of each block measured.

Presentation formats were:

- **Instructions**: instructional narrative, information and guidance provided for students.
- **Examples**: worked examples are given without any explanations.
- **Exercises**: a question or set of questions where students are informed how to do.
- **Word problems**: problems with all necessary information given for students to be able to solve the problem but they have to decide on the method.
- **Suggestions for problem solving**: students have to some extent to gather their own information before solving the problem.

The blocks were further classified with respect to content, use of diagrams, promotion of mathematical competences, and attitudes to mathematics. The two last items, mathematical competences and attitudes to mathematics, concern the new aspects of the national curricula of 1989 and 1999. The performance expectations of the 1999 curriculum cover communicating about mathematics, reflecting, explaining and using mathematical reasoning, and also solving various exercises and problems.
Furthermore the performance expectations cover linking mathematics to daily life and other activities, e.g. work, history, art and entertainment, which in this current study are measured under attitudes towards mathematics.

Content could be
- numbers,
- arithmetic, including estimation and approximation,
- ratios, proportions and percentages,
- patterns and algebra,
- geometry,
- probability and statistics, and
- other content.

Diagrams could be
- no diagrams,
- explanatory diagrams, necessary for understanding instructions or examples, and
- decorations.

Mathematical competences could be:
- *Reading and understanding*: reading and understanding symbols, explanations and worked examples.
- *Recognizing and knowing*: recognizing equivalents, representing and recalling mathematical objects and properties.
- *Using standard procedures*: using instruments, such as calculators and computers, and performing routine procedures like counting and routine computation; estimating, collecting, organizing and displaying data.
- *Reflecting on and solving problems*: formulating and clarifying problems and situations; developing strategy, solving, predicting and verifying.
- *Mathematical reasoning, communicating and explaining*: developing notation and vocabulary, developing algorithms, generalizing, conjecturing, justifying and proving; using vocabulary and notation, relating representations, describing/discussing and exploring critically.
Each block was coded for the one competence it reflected best. For a pupil to be able to show mathematical reasoning, communicate and explain he/she might also have to read and understand, use standard procedures, reflect on and solve problems but the block would be coded only as “mathematical reasoning, communicating and explaining”.

Attitudes to mathematics could be

- theoretical,
- connected to daily life,
- connected to studies and work,
- historical,
- entertaining, and
- connected to culture and arts.

Two representative examples of blocks are shown in figures 1 and 2 to illustrate the method. Each block was analysed according to area, presentation of material, content, diagrams, mathematical competence and attitude to mathematics, with results in tables 1 and 2.

Figure 1. An example of an analysis of a page in Almenn Stærðfræði I
### Table 1. Analysis of a page in Almenn stærðfræði I

<table>
<thead>
<tr>
<th>Block 1</th>
<th>Block 2</th>
<th>Block 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 measured area</td>
<td>119 cm²</td>
<td>80 cm²</td>
</tr>
<tr>
<td>2 presentation of material</td>
<td>instructions</td>
<td>example</td>
</tr>
<tr>
<td>3 content</td>
<td>arithmetic</td>
<td>arithmetic</td>
</tr>
<tr>
<td>4 use of diagram</td>
<td>no diagram</td>
<td>no diagram</td>
</tr>
<tr>
<td>5 mathematical competencies</td>
<td>reading and understanding</td>
<td>reading and understanding</td>
</tr>
<tr>
<td>6 attitudes to mathematics</td>
<td>theoretical</td>
<td>theoretical</td>
</tr>
</tbody>
</table>

### Table 2. Analysis of a page in Átta-tíu stærðfræði 2

<table>
<thead>
<tr>
<th>Block 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 measured area</td>
</tr>
<tr>
<td>2 presentation of material</td>
</tr>
<tr>
<td>3 content</td>
</tr>
<tr>
<td>4 use of diagram</td>
</tr>
<tr>
<td>5 mathematical competence</td>
</tr>
<tr>
<td>6 attitude to mathematics</td>
</tr>
</tbody>
</table>
Data handling and analysis

Validity could not be measured in any objective way but the books were all coded twice. During the first coding, problems and concerns were noted and discussed with teachers and researchers in mathematics education before a decision was made. The books were then recoded with the final coding system.

Three months after the second analysis of the books was finished, 6 chapters, 2 from each series, were randomly chosen, analysed again and the results compared with the second analysis. A reliability score, which is the fraction of cases in agreement, was 87% to 100% for different chapters with only one chapter scoring below 90%.

Results

Physical features

All the three series were intended for use during one academic year. Text I was in only one volume while Texts II and III were published in two volumes each, one for each term. The physical features of the three texts are listed in table 3.

To account for the fact that textbook pages vary considerable in size, the total area of pages in square centimetres was calculated. The total surface area is greatest for Text III as shown in table 3–9% larger than Text I and 14% larger than Text II. Text I has the greatest number of blocks and the smallest average block area. Uses of fonts were similar in all three series (Halldórsdóttir, 2008, p. 45).

Table 3. Textbook characteristics

<table>
<thead>
<tr>
<th>Book</th>
<th>Pages</th>
<th>Block area</th>
<th>Number of blocks</th>
<th>Average block area</th>
</tr>
</thead>
<tbody>
<tr>
<td>Text I Almenn stærðfræði I</td>
<td>256</td>
<td>59680 cm²</td>
<td>475</td>
<td>126 cm²</td>
</tr>
<tr>
<td>Text II Talnaspegill</td>
<td>96</td>
<td>57178 cm²</td>
<td>385</td>
<td>149 cm²</td>
</tr>
<tr>
<td>Text II Hornalina</td>
<td>104</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Text III Átta – tíu stærðfræði 1</td>
<td>112</td>
<td>64953 cm²</td>
<td>460</td>
<td>141 cm²</td>
</tr>
<tr>
<td>Text III Átta – tíu stærðfræði 2</td>
<td>112</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Presentation formats

As shown in figure 3, Text I contains most examples, 15%, while Text II and Text III contain 1% or less. Text II contains most instructions, 38%,
Comparison of three textbooks

Text I 28 %, and Text III 19 %. Text III is characterized by word problems, 57 %, while Text I had 33 % and Text II 39 %.

Use of diagrams
Use of diagrams is 36 % in Text II and 27 % in both Text I and III. The use of explanatory diagrams is similar in the texts, 16–19 %. Decorations are most often used in Text II, 19 %, while in the others the shares of diagrams are 8 % and 11 %.

Content
Figure 4 illustrates that Text III includes all content listed in the 1999 national curriculum. In Text I numbers only have sporadic coverage and in Text II geometry is well covered, but ratios, proportions, and percentages, as well as probability and statistics, are not.

Mathematical competences
A great difference in emphasis on mathematical competences was observed between the three series, see figure 5. In Text I students have to read, understand and use standard procedures most of the time. In Text II they use the above competences but are also required to reflect
Figure 4. Contents in Texts I, II and III

Figure 5. Mathematical competences in Texts I, II and III
on and solve problems, use mathematical reasoning, communicating and explaining. Text III lays the greatest emphasis on reasoning, explaining and communicating, which it also requires reading and understanding and using standard procedures as prerequisites.

**Attitudes toward mathematics**

Attitudes toward mathematics are reflected differently in the textbooks as indicated in table 4. In Text III and especially in Text II, mathematics is connected to work, culture, art, entertainment and history, while in Text I it is mainly connected to mathematical theories and daily life and only sporadically to other fields.

**Discussion**

The results of the study reveal differences of the three textbook series for grade 8 in representation, competences and attitudes toward mathematics. The two domestic series, Texts II and III, offer more word problems, suggestions for problem solving, treatment of numbers as a separate topic, emphasis on reasoning, communicating and explaining, and connections to culture and art than the translated series Text I. On the other hand, Text I provides more examples and exercises, and emphasizes training of standard procedures. The contents of Texts I and III are similar in most respects while Text I hardly treats numbers as a separate topic. Text II focuses on fewer topics.

Text I deviates in important ways from the 1989 and 1999 curricula. Mathematics is not noticeably connected to history, entertainment, culture and art, students work is homogeneous and there is little emphasis on problem solving, reasoning, explaining and communicating. Students’ work is clearly directed by information, guidance and examples. The content is according to the curriculum, except the only sporadic coverage of numbers as a separate topic.
Text II reflects in many ways the national curriculum from 1989. It differs partly from the curriculum in content. The reason may be that different books of the six-volume series were planned to emphasize different contents while the series was never completed. Mathematical competences, attitude to mathematics and diverse working methods are all in accordance with the national curriculum.

Text III reflects the 1999 curriculum well, the content is covered, mathematics is connected to history, entertainment, culture and art, and students have to reflect on and solve problems, use reasoning, explain and communicate. Text III does not steer students work as much as Text I and Text II do by information, guidance and examples.

This leads to an answer to the second purpose of the study. The results show that the two Icelandic textbook series Text II and Text III, published twenty years apart, implemented the policy, presented in the 1999 national curriculum, to a greater extent than the translated series Text I that was published before the publication of the 1989 national curriculum.

Text I is the most distant of the three texts from the intentions of the curricular reforms of 1989 and 1999. The series was published in 1987 and thus adhered in principle to the 1960 national curriculum. Text II is no longer on the market but teachers have different opinions about Texts I and III. Text III clearly adheres more closely to the national curriculum than Text I. Text I offers teachers a traditional way of teaching mathematics. A topic is presented, then problems with their solutions are demonstrated and students are given assignments working on problems that are similar to the ones that were demonstrated. Text III requires teachers to present mathematical ideas that are not detailed in the text, to create and lead discussions, promote reasoning, cooperation and problem solving. Many teachers that have used Text I are therefore required to play a substantially different role in the mathematics classroom than has been typical. The recent study by Bjarnadóttir and Hreinsdóttir (in press) points to that many teachers have found balance between the two different approaches presented in Text I and Text III and used both texts to meet the requirements of the mathematics curriculum, measured by the national tests.

Reform efforts in Icelandic mathematical education depend on official intended curriculum, textbooks as potentially implemented curriculum, and the way teachers implement the curriculum in the classroom. Given the important role of mathematics textbooks as an interpretation of policy and their close connection to classroom instructions, making a wise selection is crucial.
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**National curricula**


*Drög að námskrárm fjörir barnaskóla og gagnfræðaskóla [Drafts to national curricula for primary and lower secondary schools]*. (1948). Reykjavík: Ríkisprentsmiðjan Gutenberg.


**Textbooks researched**


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